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SYNTHESIS OF PLANAR MECHANISMS FOR PICK AND PLACE TASKS WITH GUIDING LOCATIONS

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ABSTRACT

A novel dimensional synthesis technique for solving the mixed exact and approximate motion synthesis problem for planar RR kinematic chains is presented. The methodology uses an analytic representation of the planar RR dyads rigid body constraint equation in combination with an algebraic geometry formulation of the exact synthesis for three prescribed locations to yield designs that exactly reach the prescribed pick & place locations while approximating an arbitrary number of guiding locations. The result is a dimensional synthesis technique for mixed exact and approximate motion generation for planar RR dyads. A solution dyad may be directly implemented as a 2R open chain or two solution dyads may be combined to form a planar 4R closed chain; also known as a planar four-bar mechanism. The synthesis algorithm utilizes only algebraic geometry and does not require the use of a numerical optimization algorithm or a metric on planar displacements. Two implementations of the synthesis algorithm are presented; computational and graphical construction. Moreover, the kinematic inversion of the algorithm is also included. An example that demonstrates the synthesis technique is included.

INTRODUCTION

An algorithm for synthesizing planar 4R closed-chain mechanisms, also referred to as a planar four-bar mechanisms, is presented. The planar 4R closed chain consists of four links, one link being fixed, connected in series by revolute joints (R) to form a 1 degree of freedom closed kinematic chain. A planar 4R closed chain may be viewed as the combination of two planar RR dyads; where each dyad consist of three links, one link being fixed, connect by two R joints, see Fig. 1. The synthesis objective here is to generate a planar 4R mechanism that guides a moving body exactly through two locations (referred to as *pick* & *place*) and near *n* locations (referred to as *guiding locations*). The synthesis algorithm presented utilizes an analytic representation of the planar RR dyad's rigid body constraint equation along with classical geometric constructions for exact motion synthesis. In addition a measure of the quality of the approximation to the *n* guiding locations is presented. The result is an effective synthesis procedure that is geometrically intuitive, fast to implement, and does not involve the use of any nonlinear optimization algorithms.

In related works Mirth presented an algorithm for the design of planar four-bar mechanisms based upon synthesis algorithms that yield solutions that guide a body through four locations. The synthesis of planar RR dyads for four exact locations is a well known problem whose solution involves the center point and circle point curves of classical Burmester theory [1, 2]. Mirth exploited the known properties of these curves to yield algorithms for two exact [3, 4] and three exact [5] locations and n approximate locations. Mirth solved multiple four location problems to yield multiple center or circle point curves. Graphically he identified regions of acceptable solutions

from these curves. Holte et al. extended that work with a mathematical method for identifying the regions of acceptable solutions [6]. In [7] the method was extended to include approximate velocity constraints.

Sutherland [8] presents a mixed exact-approximate synthesis technique based upon the geometric constraint of the RR dyads; that the moving pivot travel on a circle centered at the fixed pivot. He utilized the implicit algebraic representation of the circle along with an iterative solution algorithm to minimize the square of the dyad's link length; i.e. a least-squares minimization problem. Sutherland addresses the one, two, three, and four exact location problems. Kramer and Sandor [9] utilize nonlinear programming techniques to synthesis dyads that approximate the desired locations. Their selective precision methodology enables the designer to define accuracy neighborhoods around each precision location. Smaili and Dib [10] present the mixed exact-approximate synthesis of planar mechanisms for point path tasks. They use logarithmic terms in the objective function of their nonlinear optimization problem so that some points may be approximated while others are nearly reached exactly. Luu and Hayes [11] present integrated type and dimensional synthesis of planar mechanisms for rigid body guidance that uses numerical methods to determine both the mechanism type and its approximate dimensions. The methodology used here for performing the dimensional synthesis for mixed exact and approximate orientation rigid body guidance is based upon the works of [12] and [2]. The general spatial case of the methodology presented here was introduced in [13] and the spherical version was presented in [14].

This paper proceeds as follows. First, the geometry and kinematics of the planar RR dyad and the planar 4R closed chains are reviewed. Next, the synthesis algorithm for solving the mixed exact and approximate motion synthesis problem for planar RR dyads is presented. Finally, an example is presented of the application of the methodology to the synthesis of a planar four-bar mechanism to achieve two prescribed locations exactly while approximating three guiding locations.

SYNTHESIS ALGORITHM

A planar 4R closed chain is viewed as the combination of two planar RR dyads where each dyad consist of two R joints; one fixed and the other moving, see Fig. 1. The approach taken here is to separately synthesize two dyads and then join their floating links to yield a closed kinematic chain. Let the fixed axis be specified by the vector $\mathbf{u}$ measured in the fixed reference frame F and let the moving axis be specified by $\mathbf{v}$ measured in the moving frame M. Moreover, let $\mathbf{l}$ define the moving axis $\mathbf{v}$ in the fixed frame F so that, $\mathbf{l} = [A]\mathbf{v} + \mathbf{d}$ where $[A]$ is the element of SO(2) that defines the orientation of M with respect to F and $\mathbf{d}$ is the translation vector from the origin of F to the origin of M. Because the link is rigid the linear distance d between the
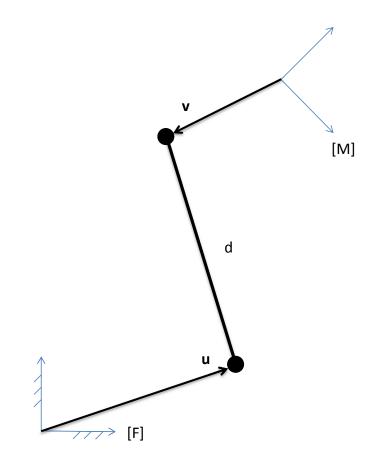

FIGURE 1. PLANAR RR DYAD GEOMETRY AND NOMENCLATURE.

two joint axes of the dyad remains constant. This geometric constraint may be expressed analytically as,

$$(\mathbf{u} - \mathbf{l}) \cdot (\mathbf{u} - \mathbf{l}) = (\mathbf{u} - [A]\mathbf{v} - \mathbf{d}) \cdot (\mathbf{u} - [A]\mathbf{v} - \mathbf{d}) = d^2. \quad (1)$$

This constraint equation is the foundation of the synthesis algorithm presented below. In order to solve the mixed exact and approximate synthesis problem we first solve the exact synthesis problem for three prescribed locations.

Exact Synthesis for Three Orientations

First, we select the moving axis $\mathbf{v}$. Second, we write Eq. 1 for each of the desired locations, $([A]_i, \mathbf{d}_i)$ $i = 1, 2, 3$. Finally, we subtract the first equation from the remaining two to arrive at a linear system of equations,

$$[P]\mathbf{u} = \mathbf{b} \quad (2)$$

where,

$$[P] = \begin{bmatrix} 2(\mathbf{l}_2^T - \mathbf{l}_1^T) \\ 2(\mathbf{l}_3^T - \mathbf{l}_1^T) \end{bmatrix},$$

$$\mathbf{b} = \begin{pmatrix} \mathbf{l}_2^T \mathbf{l}_2 - \mathbf{l}_1^T \mathbf{l}_1 \\ \mathbf{l}_3^T \mathbf{l}_3 - \mathbf{l}_1^T \mathbf{l}_1 \end{pmatrix}$$

and $\mathbf{u}$ is the desired fixed axis. We must solve Eq. 2 for each prescribed moving axis to find its corresponding fixed axis. The result is a planar RR dyad that guides a rigid body precisely through three prescribed locations.

Mixed Synthesis Algorithm

In the problem considered here we have 2 locations to reach exactly (i.e. *pick* & *place*) and n locations that serve to guide the body as it moves from the pick location to the place location.

First, it is beneficial to discuss the geometry underlying this approach. Consider the synthesis of a planar RR dyad for two exact locations for a desired moving axis $\mathbf{v}$. From Eq. 1 it is evident that a fixed axis $\mathbf{u}$ compatible with the two exact locations must be simultaneously equidistant to the moving axis in both locations; $\mathbf{l}_{pick}$ and $\mathbf{l}_{place}$. The set of all such points is a line; the perpendicular bisector of the line segment $\overline{\mathbf{l}_{pick}\mathbf{l}_{place}}$ [2].

Now consider the exact three location problem for a desired moving axis $\mathbf{v}$. The desired fixed axis $\mathbf{u}$ lies at the intersection of three perpendicular bisectors; the 1st associated with locations #1 & #2, the 2nd with locations #2 & #3, and the 3rd with #1 & #3. Generally these three lines intersect in one point; hence there is one unique fixed axis $\mathbf{u}$ associated with three planar locations and a prescribed moving axis $\mathbf{v}$.

Next we progress to the mixed synthesis algorithm. Consider a pick & place task with $n = 3$ guiding locations. Here, for simplicity, we will refer to locations by number: $1 = pick$, 2, 3, 4, and $5 = place$. Recognizing that the desired fixed pivot $\mathbf{u}$ must lie on the perpendicular bisector associated with $\mathbf{v}$ and locations #1 & #5 we proceed by solving the three exact location synthesis problem n times; once for each guiding location i.e. locations 1-2-5, 1-3-5, and 1-4-5. Each three location problem yields a fixed pivot $\mathbf{u}$ which we refer to by their associated location numbers; $\mathbf{u}_{125}$, $\mathbf{u}_{135}$, and $\mathbf{u}_{145}$. Note that each fixed pivot lies on the perpendicular bisector associated with locations #1 and #5 and therefore each solution dyad will guide the moving body exactly through locations #1 and #5. The fixed pivot $\mathbf{u}$ that will guide the moving body exactly through the pick and place locations and moves nearest the n guiding locations must lie on this perpendicular bisector. The fixed pivot $\mathbf{u}_{125}$ yields a dyad that reaches locations 1-2-5 exactly while $\mathbf{u}_{135}$ reaches locations 1-3-5 exactly and $\mathbf{u}_{145}$ reaches locations 1-4-5. Note that if the

dyad defined by $\mathbf{u}$ and $\mathbf{v}$ reached all five locations exactly then we have $\mathbf{u} = \mathbf{u}_{125} = \mathbf{u}_{135} = \mathbf{u}_{145}$. We utilize the fixed axis that lies on the perpendicular bisector associated with the two exact locations and that is nearest the perpendicular bisectors associated with the guiding locations. Hence, the solution dyad will guide the moving body exactly through the two prescribed locations and near the guiding locations for the selected moving axis. The desired fixed pivot $\mathbf{u}$ that guides the moving body exactly through the pick and place locations and moves nearest the n guiding locations is the point on the perpendicular bisector that is nearest $\mathbf{u}_{125}$, $\mathbf{u}_{135}$, and $\mathbf{u}_{145}$. See Figs. 2 & 3 which illustrates this geometric construction. Next, we discuss how we implement this synthesis algorithm computationally.

The mixed synthesis algorithm may be implemented computationally as follows. First, a desired moving axis $\mathbf{v}$ is selected. Next, we seek a corresponding fixed axis for the dyad. The fixed axis is found by solving n three location problems (Eq. 2) to yield a set of fixed axes $\mathbf{u}_i, i = 1, 2, \ldots, n$. Each of the three location problems is generated by combining the 2 exact locations (pick & place) along with 1 of the n guiding locations. Hence, we obtain n unique three location problems that are solved using Eq. 2. For the desired moving axis $\mathbf{v}$ the corresponding fixed axis $\mathbf{u}$ that guides the moving body such it exactly reaches the pick & place locations and moves nearest the guiding locations is the arithmetic mean of $\mathbf{u}_i$,

$$\mathbf{u} = \frac{\sum \mathbf{u}_i}{n}. \tag{3}$$

Step-by-Step: Geometric Construction

Here we present a *Step-by-Step* geometrical construction implementation of the synthesis algorithm.

Problem Statement: Synthesize a planar RR dyad to exactly reach pick & place locations while moving the body nearest n guiding locations.
Given: Two exact locations (pick & place), n guiding locations, and choice of moving pivot $\mathbf{v}$.
Find: The fixed pivot $\mathbf{u}$ to yield a planar RR dyad that exactly reaches the pick & place locations while moving the body nearest the n guiding locations.

1. Use triangulation to determine the position of the prescribed moving pivot $\mathbf{v}$ in each desired location: e.g. $\mathbf{l}_{pick}$, $\mathbf{l}_{place}$, $\mathbf{l}_1$, $\mathbf{l}_2$, ..., $\mathbf{l}_n$.
2. For each guiding location i solve the three location synthesis problem [2] as follows. The solution to each three location problem (i.e. the fixed pivot $\mathbf{u}_i$) is the intersection of the perpendicular bisectors of $\overline{\mathbf{l}_{pick}\mathbf{l}_i}$ and $\overline{\mathbf{l}_{pick}\mathbf{l}_{place}}$.
3. Using vector addition (*head to tail*) sum the vectors $\mathbf{u}_i, i = 1, 2, \ldots, n$. The desired fixed pivot $\mathbf{u}$ is the intersection of this vector sum and the perpendicular bisector of $\overline{\mathbf{l}_{pick}\mathbf{l}_{place}}$.

3

Step-by-Step: Computational Procedure

Here we present a *Step-by-Step* computational implementation of the synthesis algorithm.

Problem Statement: Synthesize a planar RR dyad to exactly reach pick & place locations while moving the body nearest n guiding locations.
Given: Two exact locations (pick & place), n guiding locations, and choice of moving pivot $\mathbf{v}$.
Find: The fixed pivot $\mathbf{u}$ to yield a planar RR dyad that exactly reaches the pick & place locations while moving the body nearest the n guiding locations.

1. Determine the position of the prescribed moving pivot $\mathbf{v}$ in each desired location: e.g. $\mathbf{l}_{pick}$, $\mathbf{l}_{place}$, $\mathbf{l}_1$, $\mathbf{l}_2$, ..., $\mathbf{l}_n$.
2. Solve Eq. 2 n times to yield a set of fixed axes $\mathbf{u}_i, i = 1, 2, \ldots, n$.
3. Solve Eq. 3 to yield the desired fixed axis $\mathbf{u}$.

APPROXIMATION QUALITY MEASURE

When synthesizing a mechanism it is useful to know the degree to which the n guiding locations are being reached by the planar RR dyad. We propose the following approximation quality measure $\odot$ which is based upon the structural error in each of the guiding locations.

$$\odot = 100 - \sum_{i=1}^{n} \|\mathbf{u}_i - \mathbf{u}\| \qquad (4)$$

If the dyad moves the workpiece such that it exactly reaches each of the guiding locations then each vector difference in Eq. 4 yields $\mathbf{0}$ and therefore the sum yields 0. Hence, if the dyad exactly reaches all n guiding locations the value of the approximation quality measure $\odot = 100$. This is the maximum possible value for $\odot$. Planar dyads that move the workpiece near the n guiding locations will have values of $\odot$ near 100 while those that do not approach the guiding locations well will have values of $\odot$ approaching the minimum possible value of $-\infty$.

KINEMATIC INVERSION

The above methodology may be reformulated to obtain the moving axis $\mathbf{v}$ for desired choice of fixed axis $\mathbf{u}$ by employing kinematic inversion. Inverting the relationship between the moving and fixed references frames yields the following analytical representation for the geometric constraint for a planar RR dyad [1, 2]

$$([A]^{-1}(\mathbf{u} - \mathbf{d}) - \mathbf{v}) \cdot ([A]^{-1}(\mathbf{u} - \mathbf{d}) - \mathbf{v}) = d^2. \qquad (5)$$

In the above derivations we may use Eq. 5 in place of Eq. 1 to obtain the moving axis $\mathbf{v}$ for a desired fixed axis $\mathbf{u}$.

TABLE 1.   FIVE PRESCRIBED LOCATIONS.

#	x	y	θ (deg)	Motion Type
1	0.0	0.0	0.0	exact
2	1.0	2.0	30.0	approximate
3	2.0	3.0	45.0	approximate
4	4.0	3.5	70.0	approximate
5	5.0	5.0	90.0	exact

EXAMPLE

Here we employ the preceding methodology and design a planar 4R mechanism for 5 locations; 2 exact (the starting pick location and the final place location) and three guiding locations as defined in Tab. 1 where $[A] = [Rot_z(\theta)]$. In order to prescribe the size of the coupler link and to define the attachment of the moving body to the coupler these moving axes were selected: $\mathbf{v}_a = [0 \ -1]^T$ and $\mathbf{v}_b = [1 \ -1]^T$. The mixed synthesis algorithm yielded fixed axes: $\mathbf{u}_a = [9.2662, -4.2662]^T$ and $\mathbf{u}_b = [12.3650, -3.8321]^T$. The approximation quality measure for the dyads are $\odot = 58.95$ and $\odot = 53.71$ respectively. The perpendicular bisectors and three location solutions that illustrate the application of the algorithm to determine $\mathbf{u}_a$ are shown in Fig. 2 and those associated with $\mathbf{u}_b$ are shown in Fig. 3. The resulting closed chain is a Grashof double-rocker planar four-bar mechanism that does not suffer from circuit, branch, or order defects. Its link lengths are: $input = 9.8249$, $coupler = 1.0000$, $output = 11.7125$, and $fixed = 3.1291$. The solution mechanism is shown in Figs. 4-6 with green input, yellow coupler, and red output links.

CONCLUSIONS

A novel dimensional synthesis technique for solving the mixed exact and approximate motion synthesis problem for planar RR dyads and four-bar closed kinematic chains has been presented. Moreover, an approximation quality measure that indicates the degree to which the n approximate locations have been achieved has been proposed. The methodology uses an analytic representation of the planar RR dyad's rigid body constraint equation in combination with classical geometric constructions for exact motion synthesis to yield designs that exactly reach two prescribed locations while approximating n additional guiding locations.

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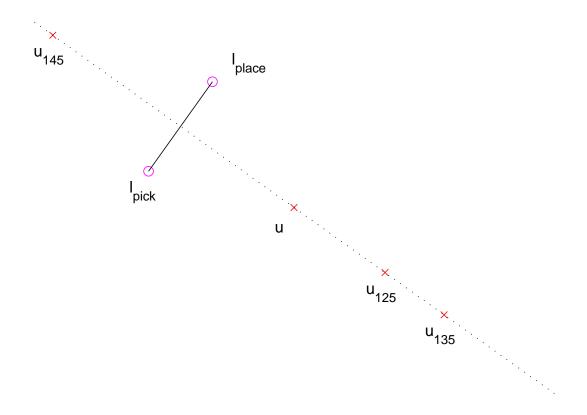[1] Bottema, O., and Roth, B., 1979. *Theoretical Kinematics*. North-Holland.

FIGURE 2. DYAD #1.

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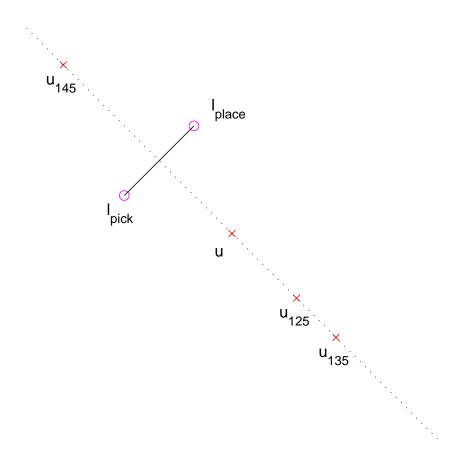[11] Luu, T., and Hayes, M. J., 2012. "Integrated type and dimensional synthesis of planar four-bar mechanisms". In

FIGURE 3. DYAD #2.

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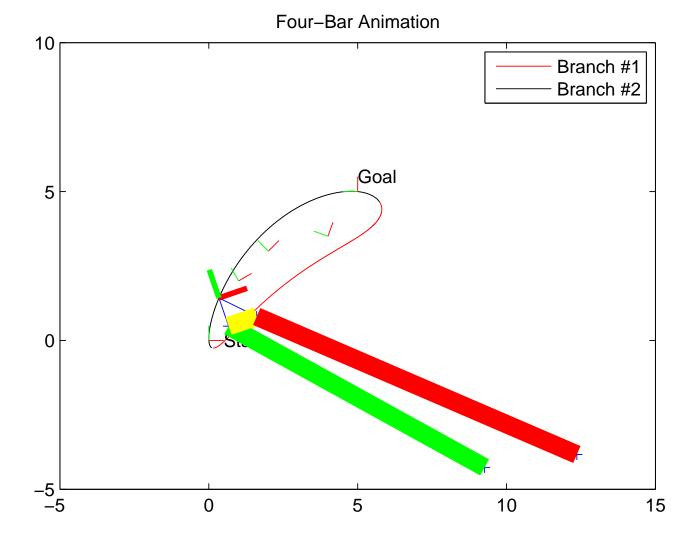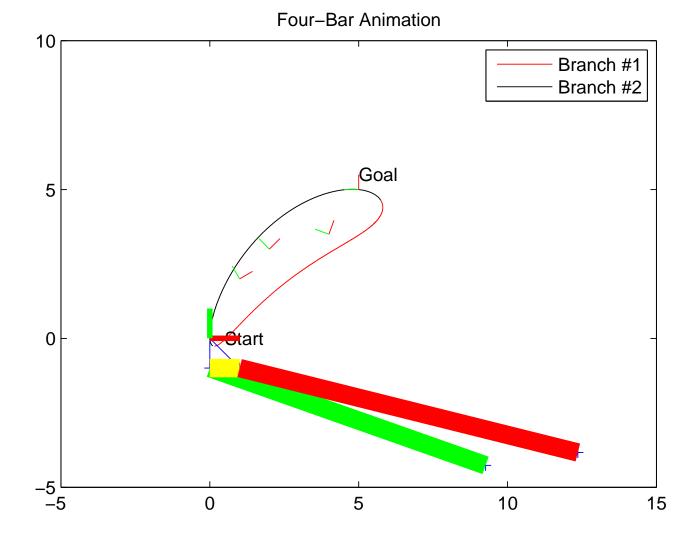
FIGURE 4.   THE SOLUTION MECHANISM.

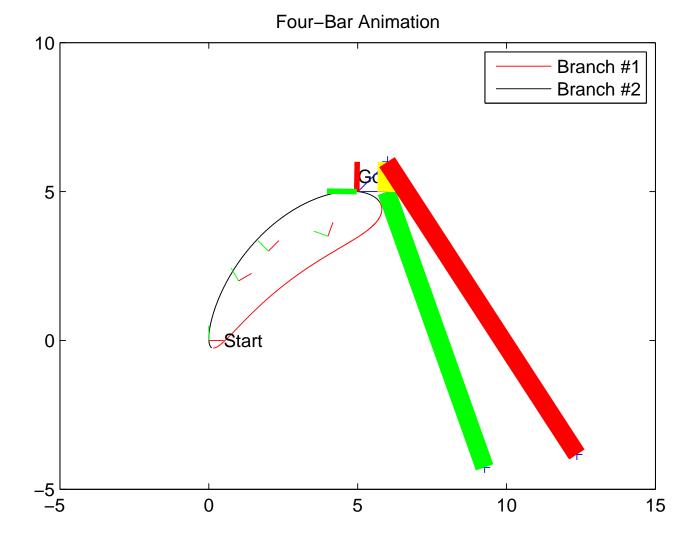**FIGURE 5**. THE MECHANISM SHOWN IN LOCATION #1.

Four−Bar Animation

FIGURE 6.   THE MECHANISM SHOWN IN LOCATION #5.